More Results on Divided Square Difference Cordial Graphs

A. Alfred Leo
Research Scholar, Research and Development centre,
Bharathiar University, Coimbatore-641046,
Tamil Nadu, India.
lee.ancy1@gmail.com

R. Vikramaprasad
Assistant Professor, Department of Mathematics,
Government Arts College, Salem-636007,
Tamil Nadu, India.

Abstract

In this article, we have further proven that some more special graphs called Subdivision graph ($K_{1,n}$), $\langle K_{1,n}{}^{(1)}, K_{1,n}{}^{(2)}\rangle$,Crown graph, Comb graph, Shell graph, Full binary tree and Triangular book are divided square difference cordial graphs.

Keywords: **Subdivision graph($K_{1,n}$), $\langle K_{1,n}{}^{(1)}, K_{1,n}{}^{(2)}\rangle$, Crown graph, Comb graph, Shell graph, Full binary tree, Triangular snake, Triangular book.**

I. INTRODUCTION

The field of Graph Theory plays an important role in various areas of pure and applied sciences. For basic notation and terminology in graph theory we refer to Bondy and Murty [4] and F. Harary [7]. In 1967, Rosa [8] introduced a labeling of G called β-valuation. A dynamic survey on different graph labeling along with an extensive bibliography was found in Gallian [6]. The concept of cordial labeling was introduced by Cahit [5]. R.Varatharajan, et.al [9] have introduced the notion of divisor cordial labeling. Also Varatharajan, et.al [10] have introduced the special classes of divisor cordial graphs. S.Abhirami, et.al [3] introduced the concept of even sum cordial labeling for some new graphs. A.Alfred Leo et.al [1] introduced the concept of divided square difference cordial labeling graphs. A.Alfred Leo et.al [2] had investigated the divided square difference cordial labeling behavior of jewel graph, $C_{n-2} + K_2$, Wheel graph, Helm graph, Flower graph, $P_n + \overline{K_m}$, $\overline{K_m} \cup P_n + 2K_1$ and Bistar.

In this paper, we have further proven that some more special graphs called Subdivision graph ($K_{1,n}$), $\langle K_{1,n}{}^{(1)}, K_{1,n}{}^{(2)}\rangle$,Crown graph, Comb graph, Shell graph, Full binary tree and Triangular book are divided square difference cordial graphs.

II. PRELIMINARIES

Definition 2.1 [6]

G*raph labeling* is an assignment of numbers to the edges or vertices or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges), then the labeling is called a vertex (edge) labeling.

Definition 2.2 [6]

A mapping $f: V(G) \to \{0,1\}$ is called *binary vertex labeling* of G and $f(V)$ is called the label of the vertex v of G under f.

Definition 2.3 [5]

A binary vertex labeling f of a graph G is called a C*ordial labeling* if $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

A graph G is cordial if it admits cordial labeling.

Definition 2.4 [1]

Let $G = (V, E)$ be a simple graph and $f: V \to \{1,2,3, \dots |V|\}$ be bijection. For each edge uv, assign the label 1 if $\left|\frac{(f(u))^2-(f(v))^2}{f(u)-f(v)}\right|$ is odd and the label 0 otherwise. f is called divided square difference cordial labeling if $|e_f(0) - e_f(1)| \le 1$, where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively.

A graph G is called divided square difference cordial if it admits divided square difference cordial labeling.

Definition 2.5 [3]

A *subdivision graph* $S(G)$ is obtained from G by subdividing each edge of G with a vertex.

Definition 2.6 [3]

Consider two graphs ${K_{1,n}}^{(1)}$ and ${K_{1,n}}^{(2)}$. Then $G = \langle {K_{1,n}}^{(1)}, {K_{1,n}}^{(2)} \rangle$ is the graph obtained by joining apex vertices of stars to a new vertex x.

Definition 2.7[3]

The *corona* $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 (with p_1 vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to all the vertices in the i^{th} copy of G_2.

Definition 2.8 [3]

The graph $P_n \odot K_1$ is called a *comb* and the graph $C_n \odot K_1$ is called a *crown*.

Definition 2.9[3]

A *shell graph* is defined as a cycle C_n with $(n-3)$ chords sharing a common end point called the apex. Shell graph are denoted as $C(n, n-3)$. A shell S_n is also called fan f_{n-1}.

Definition 2.10[3]

An *ordered rooted tree* is a binary tree if each vertex has at most two children.

Definition 2.11[3]

A *full binary tree* is a binary tree in which each internal vertex has exactly two children.

Definition 2.12[3]

One edge union of cycles of same length is called a *book*. The common edge is called the base of the book. If we consider t copies of cycles of length $n \geq 3$, then the book is denoted by ${B_n}^{(t)}$.

Definition 2.13[3]

If $n = 3$, then the book B_3 is called book with *triangular* pages.

Proposition 2.14 [1]

1. Any Path is a divided square difference cordial graph.
2. Any Cycle C_n is a divided square difference cordial graph except $n = 6, 6+d, 6+2d, \ldots$ when $d = 4$.
3. The Star graph $K_{1,n}$ is a divided square difference cordial graph.

III. MAIN RESULT

Proposition 3.1

A subdivision graph $(K_{1,n})$ is a divided square difference cordia.

Proof

Let G be a subdivision graph $(K_{1,n})$ with $|V(G)| = 2n+1$ and $|E(G)| = 2n$.
Let $u, v_1, v_2, \ldots, v_n, v_1', v_2', \ldots, v_n'$ are the vertices of subdivision graph $(K_{1,n})$.
Define a map $f: V(G) \to \{1,2, \ldots, 2n+1\}$. Now we can label the vertices as

$$f(u) = 2n+1, f(v_i) = 2i, f(v_i') = 2i-1, \ 1 \leq i \leq n$$

Then, we get $e_f(0) = e_f(1)$.
Thus $|e_f(0) - e_f(1)| \leq 1$.
Hence G is a divided square difference cordial graph.

Example 3.2

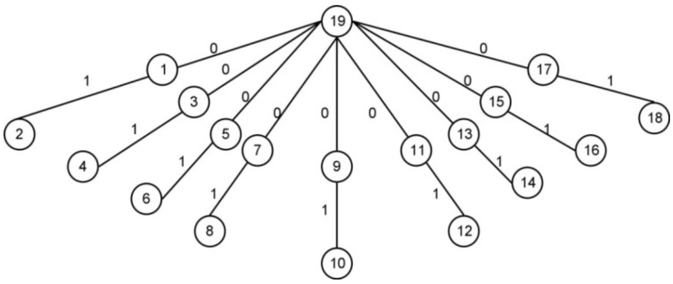

Fig 1. Subdivision graph $(K_{1,9})$

Proposition 3.3

The graph $\langle K_{1,n}{}^{(1)}, K_{1,n}{}^{(2)} \rangle$ is a divided square difference cordial.

Proof

Let G be a graph $\langle K_{1,n}{}^{(1)}, K_{1,n}{}^{(2)} \rangle$ with $|V(G)| = 2n + 3$ and $|E(G)| = 2n + 2$.
Let $u_1, u_2, \dots, u_n$ are the vertices of $K_{1,n}{}^{(1)}$ and $v_1, v_2, \dots, v_n$ are the vertices of $K_{1,n}{}^{(2)}$.
Let u and v are the apex vertices of $K_{1,n}{}^{(1)}$, $K_{1,n}{}^{(2)}$ respectively which are adjacent to a common vertex x.
Define a map $f: V(G) \rightarrow \{1,2, \dots, 2n + 3\}$.
We can label the vertices by taking $f(x) = 2n + 3, f(u) = 1, f(u_i) = i + 1, f(v) = n + 2, f(v_i) = n + 2 + i, 1 \leq i \leq n$.
Then, we get $e_f(0) = e_f(1)$.
Therefore $|e_f(0) - e_f(1)| \leq 1$.
Hence G is a divided square difference cordial graph.

Example 3.4

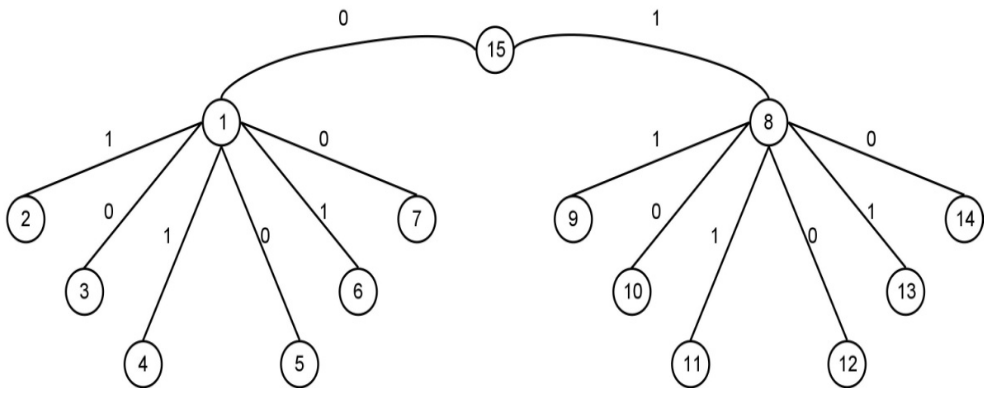

Fig 2. Graph $\langle K_{1,6}{}^{(1)}, K_{1,6}{}^{(2)} \rangle$

Proposition 3.5

The Crown graph $C_n \odot K_1$ is a divided square difference cordial.

Proof

Let G be a Crown graph $C_n \odot K_1$ with $|V(G)| = 2n = |E(G)|$.
Let $u_1, u_2, \dots, u_n$ are the vertices of cycle C_n and $v_1, v_2, \dots, v_n$ are the vertices of n copies of K_1.
Define a map $f: V(G) \rightarrow \{1,2, \dots, 2n\}$.
First we can construct the cycle C_n by Proposition 2.14 (2).
Then we will assign the label values for the vertices v_i by taking $f(v_i) = n + i, 1 \leq i \leq n$.
Then, we get $|e_f(0) - e_f(1)| \leq 1$.
Hence G is a divided square difference cordial graph.

Example 3.6

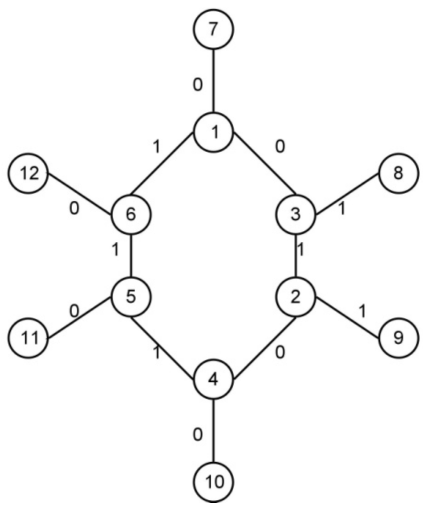

Fig 3. Crown graph $C_6 \odot K_1$

Proposition 3.7

The Comb graph $P_n \odot K_1$ is a divided square difference cordial.

Proof

Let G be a Comb graph $P_n \odot K_1$ with $|V(G)| = 2n$ and $|E(G)| = 2n - 1$.
Let $u_1, u_2, \dots, u_n$ are the vertices of n copies of K_1 and $v_1, v_2, \dots, v_n$ are the vertices of path P_n.
Define a map $f: V(G) \to \{1,2, \dots, 2n\}$.
First we can construct the path P_n by Proposition 2.14 (1).
Then we will assign the label values for u_i by taking $f(u_i) = n + i, 1 \le i \le n$.
Then, we get $|e_f(0) - e_f(1)| \le 1$.
Hence G is a divided square difference cordial graph.

Example 3.8

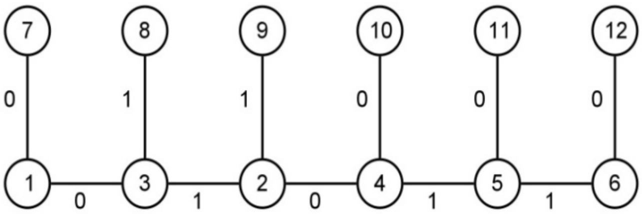

Fig 4. Comb graph $P_6 \odot K_1$

Proposition 3.9

The Shell graph $C(n, n-3)$ is a divided square difference cordial except for $n = 6, 6+d, 6+2d, \dots$ where $d = 4$.

Proof

Let G be a Shell graph $C(n, n-3)$ with $|V(G)| = n$ and $|E(G)| = 2n - 3$. In the graph $C(n, n-3)$ we will get $n - 3$ chords.
Let $v_1, v_2, \dots, v_n$ are the vertices of the cycle C_n. Now, we define a map $f: V(G) \to \{1,2, \dots, n\}$.
First we can construct the cycle C_n by Proposition 2.14(2). Then choose v_1 as the apex vertex and join it to all the non-adjacent vertices of the cycle C_n by $n - 3$ edges.
Thus, $e_f(1) = e_f(0) + 1$
Then, we get $|e_f(0) - e_f(1)| \le 1$.
Hence G is a divided square difference cordial graph.

Example 3.10

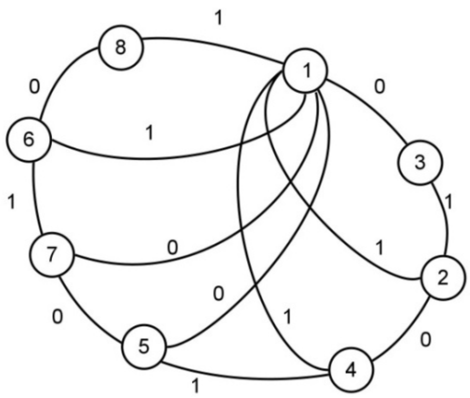

Fig 5. Shell graph $C(8,5)$

Proposition 3.11

Every full binary tree is a divided square difference cordial.

Proof

Let T be a full binary tree. Let u be a root of T which is called zero level vertex. The j^{th} level of T has 2^j vertices. If T has n levels, then $|V(T)| = 2^{n+1} - 1$ and $|E(G)| = 2^{n+1} - 2$.
Now, we can assign $2^{n+1} - 1$ vertices as follows. Here j denotes the level of T.

Level	Vertex label	Range
$j = 1$	$f(u_i) = 2^{n+1} - (i + 1)$	$1 \le i \le 2^j$
$j = 2$	$f(u_i) = 2^{n+1} - (i + 3)$	$1 \le i \le 2^j$
$j = 3$	$f(u_i) = 2^{n+1} - (i + 7)$	$1 \le i \le 2^j$
$j = 4$	$f(u_i) = 2^{n+1} - (i + 15)$	$1 \le i \le 2^j$
.	.	.
.	.	.
.	.	.
$j = n$	$f(u_i) = 2^{n+1} - \left(i + (2^j - 1)\right)$	$1 \le i \le 2^j$

Thus, we have $e_f(0) = e_f(1) = 2^n - 1$
Then, we get $|e_f(0) - e_f(1)| \le 1$.
Hence T is a divided square difference cordial graph.

Example 3.12

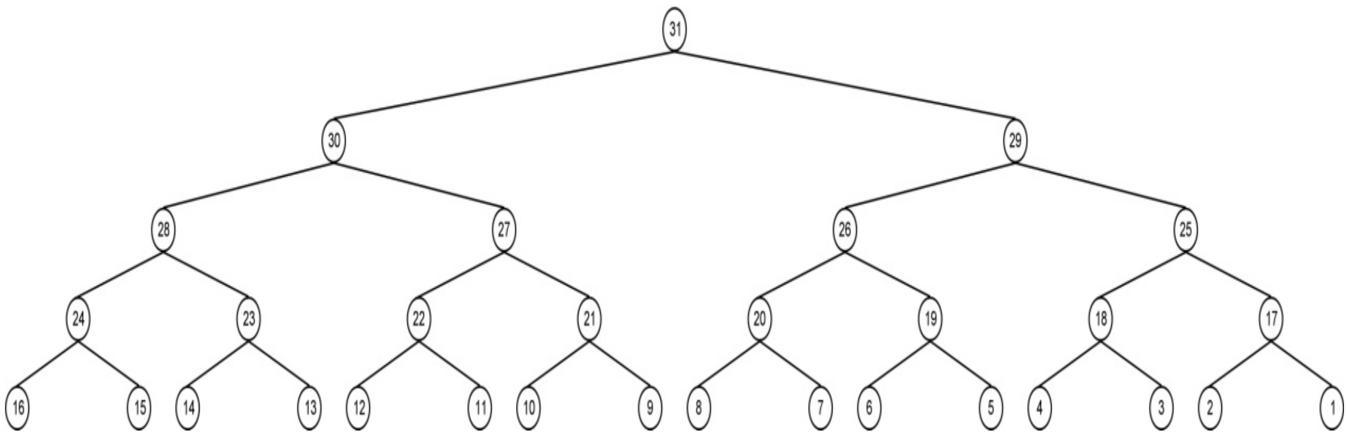

Fig 6. Full binary tree with 4 levels

Proposition 3.13

A book B with triangular pages is a divided square difference cordial.

Proof

Let B be a book with triangular pages. Let v_1, v_n are the vertices of the common edge of the triangular pages and let $v_2, v_3, \dots, v_{n-1}$ are the other end of the triangle. Here B has $2n - 3$ edges.
The edge set is given by $E(G) = \{v_1 v_i, v_n v_i, v_1 v_n, 2 \le i \le n - 1\}$
Now, define a map $f: V(G) \to \{1,2, \dots, n\}$.
We can label the vertices by taking,

$$f(v_1) = 1, f(v_n) = n, \ f(v_i) = i, 2 \le i \le n - 1$$

Thus $e_f(0) = n - 2$ and $e_f(1) = n - 1$
Then we get $|e_f(0) - e_f(1)| \le 1$.
Hence G is a divided square difference cordial graph.

Example 3.14

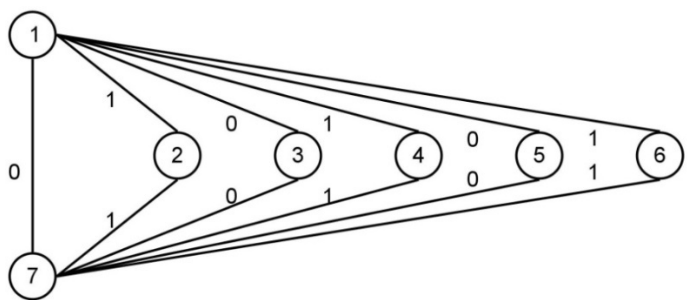

Fig 7. Book B with triangular pages of 7 vertices

IV. CONCLUSION

In this paper, the concepts of divided square difference cordial labeling behavior of Subdivision graph $(K_{1,n})$, $\langle K_{1,n}{}^{(1)}, K_{1,n}{}^{(2)}\rangle$,Crown graph, Comb graph, Shell graph, Full binary tree, Triangular book were discussed.

ACKNOWLEDGMENT

The authors are highly thankful to the anonymous referees for constructive suggestions and comments. Also thankful to Dr.R.Dhavaseelan for his motivation, valuable feedback and comments.

REFERENCES

[1] *A.Alfred Leo, R.Vikramaprasad and R.Dhavaseelan; Divided square difference cordial labeling graphs, International Journal of Mechanical Engineering and Technology,* ***9****(1), pp.1137 – 1144, Jan 2018.*

[2] *A.Alfred Leo and R.Vikramaprasad; Divided square difference cordial labeling of some special graphs, Submitted.*

[3] *S.Abhirami, R.Vikramaprasad and R.Dhavaseelan; Even sum cordial labeling for some new graphs, International Journal of Mechanical Engineering and Technology,* ***9****(2), pp.214 – 220, Feb 2018.*

[4] *Bondy J.A and Murty.U.S.R, Graph theory and applications, North Holland, New York, 1976.*

[5] *I. Cahit, "Cordial graphs: a weaker version of graceful and harmonious graphs," Ars Combinatoria, 23, pp. 201 – 207,1987.*

[6] *J. A. Gallian, A dynamic survey of graph labeling, Electronic J. Combin.* ***15,****DS6, pp.1 – 190, 2008.*

[7] *F. Harary, Graph theory, Addison-Wesley, Reading, MA.,1969.*

[8] *A. Rosa, On certain valuations of the vertices of a graph, Theory of Graphs (Internat. Symposium, Rome, July 1966), Gordon and Breach, N. Y. and Dunod Paris , pp.349 – 355,1967*

[9] *R.Varatharajan, S.Navaneethakrishnan and K.Nagarajan,Divisor cordial graphs, International J.Math. Combin, Vol.4, 15-25,2011.*

[10] *R.Varatharajan, S.Navaneethakrishnan, K.Nagarajan, Special classes of divisor cordial graphs, International Mathematical forum, Vol.7, No.35, pp.1737 –1749,2012.*